COMPARISON OF RADIAL AND FRONTAL BLOWING OVER A ROTATING SCREENED DISK

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The results are presented of computer calculations of the flow of a viscous fluid between a cooled rotating disk and a fixed screen for two methods of supplying the cooling medium—radial and frontal.

A number of methods exist for cooling the disks of gas turbines, including radial blowing of the cooling medium into the gap between disk and screen, frontal ventilation of the disk with medium supplied through the screen, a combination of these methods with water cooling of the screen, etc. The results of these investigations have been described by a number of authors [1–9].

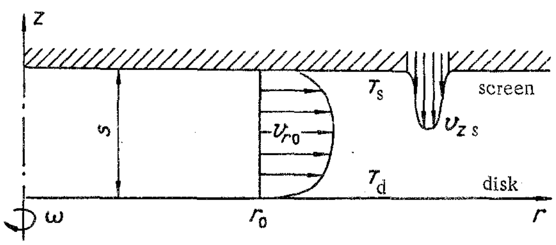

Fig. 1. Diagram of the problem.

We shall make a comparison of these methods of cooling on the basis of solution of the Navier-Stokes and energy equations for small relative values of the gap, s/r_0, between disk and screen (Fig. 1). To compare heat transfer conditions, we shall choose the case where the disk and screen surfaces are isothermal, this being optimal from the viewpoint of thermal stress.

At small s/r_0, which corresponds to actual situations, the dominant viscous terms in the flow and energy equations will be those derivatives with respect to z, and the equations may be put in the form [9]

$$\mathrm{Re}^2 u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial y} - \frac{v^2}{x} = -\frac{d\pi^0}{dx} + \frac{\partial^2 u}{dy^2}, \tag{1}$$

$$\mathrm{Re}^2 \left(u \frac{\partial v}{\partial x} + \frac{uv}{x} \right) + w \frac{\partial v}{\partial y} = \frac{\partial^2 v}{\partial y^2}, \tag{2}$$

$$\mathrm{Re}^2 \left(\frac{\partial u}{\partial x} + \frac{u}{x} \right) + \frac{\partial w}{\partial y} = 0, \tag{3}$$

$$\mathrm{Re}^2 \mathrm{Pr}\, u \frac{\partial \Theta}{\partial x} + \mathrm{Pr}\, w \frac{\partial \Theta}{\partial y} = \frac{\partial^2 \Theta}{\partial y^2}, \tag{4}$$

where the pressure in the gap is constant, i.e., $\partial\pi^0/\partial y = 0$. Here

$$x = r/r_0,\ y = z/s,\ v = v_\varphi/r_0\omega,\ w = v_z s/\nu,\ u = v_r/(r_0\omega\,\mathrm{Re}),$$

$$\pi^0 = p/\rho(r_0\omega)^2,\ \Theta = (T - T_s)/(T_d - T_s),$$

$$\mathrm{Re} = s^2\omega/\nu,\ \mathrm{Pr} = \mu c_p/\lambda. \tag{5}$$

Dissipation is not included in the energy equation.

Thus the initial system of nonlinear equations (Navier-Stokes) of elliptic type is transformed into the system (1), (2) of parabolic equations; it is therefore necessary to exclude cases of reverse flow in the gap. The calculations of [9] show good agreement between solutions of (1)–(4) and those of the full equations in the absence of reverse flow.

The boundary conditions of the problem are: at the initial radius x = 1

$$u = u^0(y),\ v = v^0(y),\ \Theta = \Theta^0(y), \tag{6}$$

on the rotating disk y = 0

$$u = w = 0,\ v = x,\ \Theta = 1, \tag{7}$$

on the fixed screen y = 1

$$u = v = \Theta = 0,\quad w = w_M. \tag{8}$$

The method of numerical solution of the boundary problem is based on the fact that each equation of the second order system (1)–(4) has the form

$$a\frac{\partial f}{\partial x} + b\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial y^2} + c, \tag{9}$$

where a, b, c depend on the unknown functions, and may be approximated with the aid of the two-layer implicit six-point difference scheme:

$$\begin{aligned} & a_m^{n-1/2}\frac{f_m^n - f_m^{n-1}}{\Delta x} + \\ & + b_m^{n-1/2}\frac{\sigma(f_{m+1}^n - f_{m-1}^n) + (1-\sigma)(f_{m+1}^{n-1} - f_{m-1}^{n-1})}{2\Delta y} = \\ & = \frac{1}{(\Delta y)^2}\{[(1-\sigma)(f_{m+1}^{n-1} - f_m^{n-1}) + \sigma(f_{m+1}^n - f_m^n)] - \\ & - [(1-\sigma)(f_m^{n-1} - f_{m-1}^{n-1}) + \sigma(f_m^n - f_{m-1}^n)]\} + c_m^{n-1/2}, \end{aligned} \tag{10}$$

where the values of the desired mesh functions f_m^n were determined on the rectangular net

$$x = x_0 + n\Delta x,\quad y = m\Delta y$$

$$(n = 0, 1, 2, \ldots;\ m = 0, 1, \ldots, M)$$

and coefficients a, b, and c—at the "half-integral" points

$$x = x_0 + n\Delta x,\quad y = (m + 1/2)\Delta y,$$

$$x = x_0 + (n + 1/2)\Delta x,\quad y = m\Delta y$$

σ being the averaging parameter, $1/2 \le \sigma \le 1$, $x_0 = 1$.

At each layer n we obtain a difference equation of second order

$$\alpha_m^n f_{m-1}^n + \beta_m^n f_m^n + \gamma_m^n f_{m+1}^n = \delta_m^n,$$

which is solved by a run at the assigned boundary values of f_0^n and f_m^n. The coefficients α, β, γ, δ depend not only on the values of the unknown functions in the preceding layer, but also on the values in the current layer, and so the calculation proceeds layer by layer, starting with n = 0 and iterating. The values of the functions at the "half-integral" points are determined from their values on the main net according to the formula

$$f_m^{n-1/2} = \frac{1}{2}(f_m^n + f_m^{n-1}).$$

The pressure gradient $t = d\pi^0/dx$ is determined from the conditions specified at the boundaries y = 0 and y = 1 for the transverse velocity component ω, and bringing in the continuity equation (3) in the difference form

$$\mathrm{Re}^2\left[\frac{u_m^n - u_m^{n-1}}{2\Delta x} + \frac{u_{m+1}^n - u_{m+1}^{n-1}}{2\Delta x} + \frac{1}{1+(n-1/2)\Delta x}u_{m+1/2}^{n-1/2}\right] + \frac{w_{m+1}^{n-1/2} - w_m^{n-1/2}}{\Delta y} = 0, \qquad (11)$$

where

$$u_{m+1/2}^{n-1/2} = \frac{1}{4}\left(u_{m+1}^n + u_{m+1}^{n-1} + u_m^n + u_m^{n-1}\right).$$

The solution u_m^n of the corresponding difference equation

$$\alpha_m^n u_{m-1}^n + \beta_m^n u_m^n + \gamma_m^n u_{m+1}^n = \delta_m^n + t^{n-1/2} \qquad (12)$$

is put in the form

$$u_m^n = u_m^{n0} + u_m^{n1} t^{n-1/2},$$

where u_m^{n0} is the solution of (12) with $t^{n-1/2} = 0$, and u_m^{n1} the solution with $\delta_m^n = 0$, $t^{n-1/2} = 1$. Then the desired $t^{n-1/2}$ is determined from u_m^{n0}, u_m^{n1}, and the boundary conditions $\omega_0^{n-1/2}$, $\omega_M^{n-1/2}$ in accordance with (11). Detailed calculations are given in [9]. The calculation was performed on the "Ural-2" computer.

Let us examine the results of the calculations. They were performed with $\Delta x = 0.005$, $\Delta y = 0.025$, and an iteration accuracy of $\varepsilon = 10^{-4}$–10^{-5}. As the Reynolds number is increased with fixed radial blowing ($\varphi_r = 4.64$), the pressure gradient increases (see table), while the friction stress at the walls and the heat transfer coefficient at the screen decrease. The nature of the flow and heat transfer in the gap is illustrated in Fig. 2. In this the distribution of circumferential velocities at the initial radius was chosen close to linear, while the radial velocities were constant in the gap $v_r = v_{r0}$, except for the wall regions, and the temperature equalled that of the cold screen $\Theta^0 \to 0$, except for the regions near the disk, where $\Theta^0 \to 1$ when $y \to 0$.

At large Re numbers reverse flow appears at the fixed screen ($\tau_r|_s < 0$): for Re = 50 ($\varphi_r = 4.64$) with $x \approx 1.67$ (but the computer continued the calculation), and for Re = 100 with $x \approx 1.27$ (here the computer stopped, as soon as $\tau_r|_d < 0$ at the disk with $x \approx 1.37$).

Heat transfer on the rotating disk increases with increase of Re, but if the air is heated uniformly at the initial radius, i.e., the temperature distribution is linear ($\Theta^0 \approx 1 - y$), there is no increase, and, on the contrary, the heat transfer even decreases with increase of Re.

With increase of mass flow rate φ_r the friction stress τ_r at the screen increases, as it does at the disk, i.e., the reverse flows are suppressed. Radial blowing is therefore used in gas turbines to avoid indraft of hot gas from the inter-crown gap into the gap between disk and screen. As φ_r increases, so also do the pressure gradient, friction stress, and heat transfer (see the table), but, as was the case above, when there is uniform heating at the initial radius ($\Theta^0 \approx 1 - y$), the cooling effect decreases sharply and diminishes with increase of φ_r.

Let us compare the heat transfer intensity for radial blowing and for uniform frontal blowing ($w_M = \mathrm{const} < 0$), with the same amount of cooling air and the same temperature conditions at the initial radius. The calculation for Re = 25 and $\varphi_r = Q_T = 4$, i.e., $(w_z)_M/2s\omega = -1$, indicates the advantage of frontal blowing over radial blowing (Fig. 3).

The mean heat transfer coefficient between x = 1 and x = 2 near the rotating disk for radial blowing ($(\mathrm{Nu})_m = 2.86$) is considerably less than the corresponding coefficient $\varphi_r = 4.64$ for frontal blowing.

Frontal blowing is accomplished in practice not by uniform supply of the medium, but by means of a concentrated jet through an annular orifice (or an annular system of individual jets) in conjunction with radial blowing; calculations were therefore performed for these cases (Fig. 4). The intensity of cooling increased more rapidly when annular blowing was introduced than at the same flow rate of cooling air in radial blowing (see the table); the addition of annular blowing with $\varphi_T = 3.12$ to $\varphi_T = 4.64$ gives a mean heat transfer coefficient increment $\Delta(\mathrm{Nu})_m = 1.11$, greater than for a large increase of radial blowing $\Delta\varphi_r = 4.64$, when $\Delta(\mathrm{Nu})_m = 0.73$. We note that the blowing velocity distribution $w_M(x)$ for $\varphi_T = 3.12$ was determined by interpolation between the values

$x =$	1.45	1.5	1.55	1.6	1.65	1.7	1.75	1.8	1.85	1.9
$-w_M =$	0	6.25	25	56.25	100	100	56.25	25	6.25	0

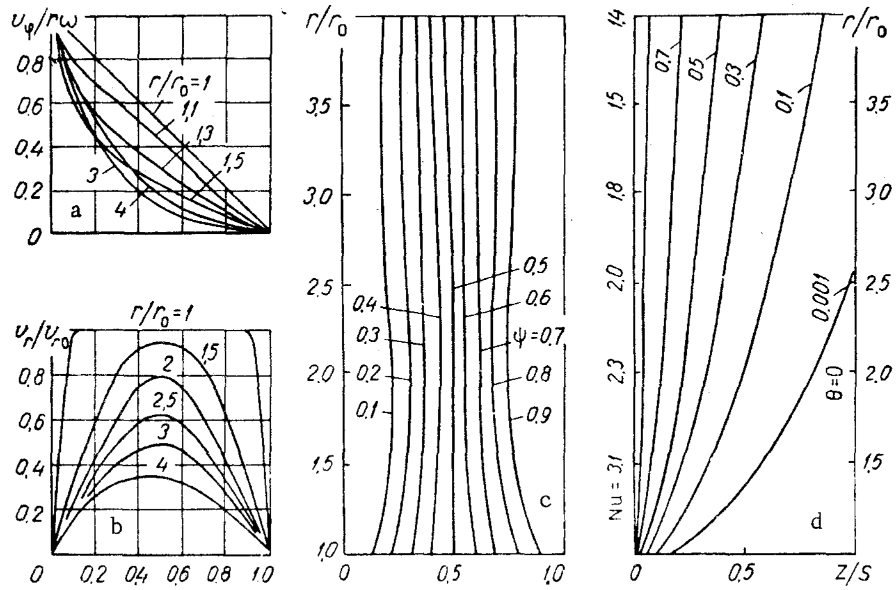

Fig. 2. Nature of flow and heat transfer for air (Pr = = 0.7) with radial blowing; Re = 10, φ_r = 4.64, u^0 = const, $v^0 \approx 1 - y$, $\Theta^0 \approx 0$: a) variation of circumferential velocity distribution along the radius; b) variation of radial velocity distribution along the radius; c) stream lines ψ = const in the gap between the rotating disk (z = 0) and the fixed screen (z = s); d) isotherms Θ = const in the gap between the disk and the screen (local values of Nu are shown on the disk surface z = 0).

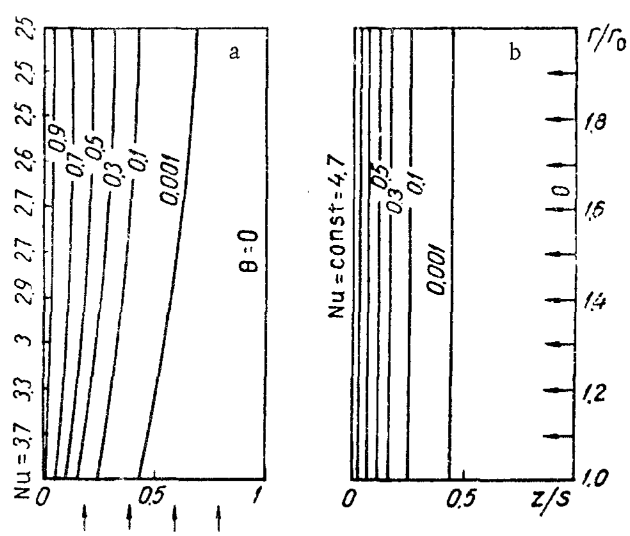

Fig. 3. Comparison of heat transfer conditions under radial blowing and uniform frontal blowing at the same flow rate of cooling air (Pr = 0.7) Re = 25, $\varphi_r = \varphi_T$ = 4: a) isotherms Θ = const with radial blowing; b) isotherms Θ = const with uniform frontal blowing (local Nu numbers on the disk surface are shown).

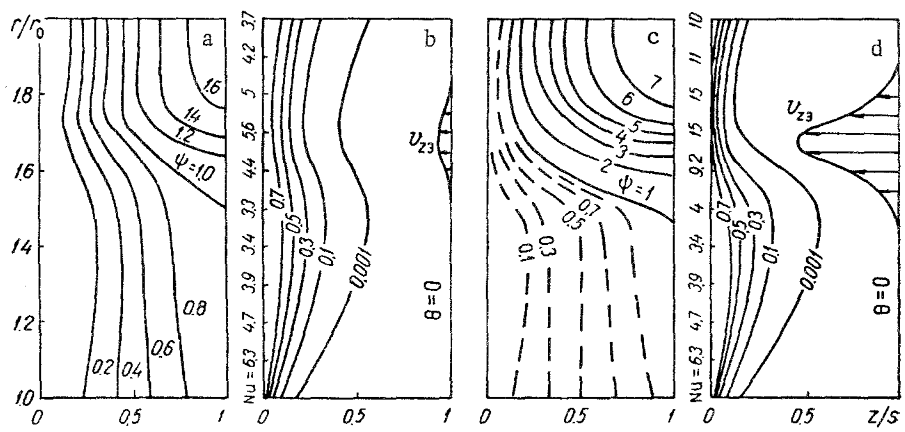

Fig. 4. The influence of frontal annular blowing with Re = 10, $\varphi_r = 4.64$, Pr = 0.7, $v^0 \approx 1 - y$, $u^0 \approx$ const, $\Theta^0 \approx 0$, $\varphi_T =$ = 3.12 (a, b) and 31.2 (c, d): a, c) ψ = const lines in the gap between rotating disk and fixed screen; b, d) isotherms Θ = const in the gap between disk and screen (local Nu values at the disk surface z = 0 are shown).

Values of the Flow and Heat Transfer Parameters with $r/r_0 = 2$ ($Pr = 0.7$)

Re	φ_r	φ_T	$\frac{d\pi^0}{dx}$	$(\bar{\tau}_r)_d$	$(\bar{\tau}_r)_s$	$(\bar{\tau}_\varphi)_d \times 10$	$(\bar{\tau}_\varphi)_s \times 10$	$(Nu)_d$ at $\theta^\circ \approx 0$	$(Nu_{cp})_d$ at $\theta^\circ \approx 0$	$(Nu)_s$ at $\theta^\circ \approx 0$	$(Nu)_d$ at $\theta^\circ \approx 1-y$	$(Nu)_s$ at $\theta^\circ \approx 1-y$
Influence of Reynolds number												
1	4.64	0	−23.2	13.6	13.3	33.1	11.6	1.05	1.55	0.949	1.00	1.00
10	4.64	0	1.87	0.997	0.799	6.66	0.408	2.34	3.72	0.026	0.815	0.835
50	4.64	0	1.92	0.214	−0.059	2.42	0.042	4.47	5.91	0	0.769	0.459
Influence of radial blowing												
10	0.928	0	−0.146	0.438	0.192	4.57	0.725	1.37	2.20	0.683	1.08	0.942
10	2.32	0	−0.065	0.715	0.492	5.74	0.486	1.92	3.02	0.212	1.02	0.963
10	9.28	0	8.99	1.27	1.20	8.45	0.318	2.77	4.45	0	0.739	0.753
Influence of additional frontal blowing												
10	4.64	3.12	2.64	3.82	0.710	10.4	0	3.72	4.83	0	—	—
10	4.64	31.2	27.7	43.6	1.21	26.0	0	9.55	8.95	0	—	—
10	0.928	31.2	23.8	36.9	1.25	27.0	0	9.47	8.27	0	—	—

(for $1 \le x \le 1.45$, $1.9 \le x \le 2.0$, $w_M = 0$), and for $\varphi_T = 31.2$ all these values were increased by a factor of 10.

Concentrated annular blowing also increases the pressure gradient and the friction stresses at the walls.

Finally, the results of calculations are presented to determine the influence of Pr number on heat transfer character in the radial blowing case. With $r/r_0 = 2$ for Re = 10, $\varphi_r = 4.64$ and $\Theta^0 \approx 2$ we have

Pr	$(Nu)_d$	$(Nu)_s$
0.3	1.73	0.292
0.7	2.34	0.026
3	3.73	0
10	5.20	0
50	7.03	0

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NOTATION

v_r, v_φ, v_z—radial, circumferential, and transverse velocity vector components, respectively; p—pressure; T—temperature; ρ—density; ν—kinematic viscosity; $\mu = \rho\nu$; ω—angular velocity; r_0—initial radius; s—gap width; $\varphi_r = G_r/2\pi\rho r_0^2 s\omega$; $\varphi_T = G_T/2\pi\rho r_0^2 s\omega$; $G_r = 2\pi\rho r\int_0^s v_{r0}\,dz$; $G_T = 2\pi\rho\int_0^r r|v_{zs}|\,dr$, $\bar{\tau}_r = \tau_r/\rho r_0 s\omega^2 = \frac{\partial u}{\partial y}$; $\bar{\tau}_\varphi = \tau_\varphi/\rho r_0 s\omega^2 = \frac{1}{Re}\frac{\partial v}{\partial y}$; $Nu = qs/\lambda(T_d - T_s) = \frac{\partial\Theta}{\partial y}$; ψ—stream function; q—specific heat flux. Subscripts: s—on the screen; d—on the disk.

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